Answer-Keys to Even Review Problems for Test 2

Section 7.3

2. $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}\,dx = 8\int \tan^3\theta\,\sec\theta\,d\theta = \frac{1}{3}(x^2+4)^{3/2} - 4\sqrt{x^2+4} + C$

7. $\displaystyle\int_0^2 \frac{1}{(x^2+4)^{3/2}}\,dx = \frac{1}{4}\int_0^{\pi/4} \cos\theta\,d\theta = \frac{1}{4\sqrt{2}}$

8. $\displaystyle\int \frac{1}{t^2\sqrt{t^2-16}}\,dt = \frac{1}{16}\int \cos\theta\,d\theta = \frac{\sqrt{t^2-16}}{16t} + C$

14. $\displaystyle\int_0^1 \frac{1}{(x^2+1)^2}\,dx = \int_0^{\pi/4} \cos^2\theta\,d\theta = \frac{\pi}{8} + \frac{1}{4}$

Section 7.4

4a. $\displaystyle x^2 + \frac{A}{x-1} + \frac{B}{(x-1)^2}$

20.; $\displaystyle\int_2^3 \left[\frac{x(3-5x)}{(3x-1)(x-1)^2}\right] dx = \int_2^3 \left(\frac{1}{3x-1} + \frac{-2}{x-1} + \frac{-1}{(x-1)^2}\right) dx = -\ln 2 - \frac{1}{3}\ln 5 - \frac{1}{2}$

22. $\displaystyle\int \left[\frac{x^4+9x^2+x+2}{(x^2+9}\right] dx = \int \left(x^2 + \frac{x}{x^2+9} + \frac{2}{x^2+9}\right) dx = \frac{x^3}{3} + \frac{1}{2}\ln(x^2+9) + \frac{2}{3}\tan^{-1}\frac{x}{3} + C$

Section 7.8

1. (a) Improper integral as $\dfrac{x}{x-1}$ has infinite discontinuity at $x=1$ and $\displaystyle\int_1^2 \frac{x}{x-1}\,dx = \lim_{R\to 1^+}\int_R^2 \frac{x}{x-1}\,dx$

(b) Infinite interval of integration so improper integral and $\displaystyle\int_0^\infty \frac{1}{1+x^3}\,dx = \lim_{R\to\infty}\int_0^R \frac{1}{1+x^3}\,dx$

(c) Infinite interval of integration so improper integral

and $\displaystyle\int_{-\infty}^{\infty} x^2e^{-x^2}\,dx = \lim_{R\to-\infty}\int_R^0 x^2e^{-x^2}\,dx + \lim_{b\to\infty}\int_0^b x^2e^{-x^2}\,dx$

(d) Improper integral as $\cot x$ has infinite discontinuity at $x=0$ and $\displaystyle\int_0^{\pi/4} \cot x\,dx = \lim_{R\to 0^+}\int_R^{\pi/4} \cot x\,dx$

2. (a) proper integral (b) Improper integral as $\tan x$ has infinite discontinuity at $x = \frac{\pi}{2}$

and $\displaystyle\int_0^{\pi} \tan x\,dx = \lim_{b\to(\frac{\pi}{2})^-} \int_0^{b} \tan x\,dx + \lim_{R\to(\frac{\pi}{2})^+} \int_R^{\pi} \tan x\,dx$

(c) Improper integral as $\dfrac{1}{x^2-x-2} = \dfrac{1}{(x-2)(x+1)}$ has infinite discontinuity at $x=-1$

and $\displaystyle\int_{-1}^{1} \frac{1}{x^2-x-2}\,dx = \lim_{R\to(-1)^+} \int_R^{1} \frac{1}{x^2-x-2}\,dx$

(d) Infinite interval of integration so improper integral and $\displaystyle\int_0^{\infty} e^{-x^3}\,dx = \lim_{b\to\infty} \int_0^{b} e^{-x^3}\,dx$

50. Integral diverges by comparison with $g(x) = \dfrac{1}{\sqrt{x}}$

Section 11.1

26. Converges to 2 **28.** Converges to 3 **30.** Converges to 0

32. Converges to -1 **36.** Diverges, limit dne **38.** Converges to 1

40. Converges to 0 **42.** Converges to 0 **48.** Converges to 1

50. Converges to 0 **56.** Converges to 0

Section 11.2

4. Series converges and Sum$= \displaystyle\lim_{n\to\infty} s_n = \frac{1}{4}$ **22.** Series converges and Sum$= \dfrac{5}{\pi - 1}$

24. Series diverges **26.** Divergent

46. $S_n = \dfrac{1}{\sqrt{4}} - \dfrac{1}{\sqrt{n+1}}$ and Sum$= \displaystyle\lim_{n\to\infty} S_n = \frac{1}{2}$. Series converges.

60. Series converges for $\dfrac{19}{4} \le x \le \dfrac{21}{4}$ ans Sum$= \dfrac{1}{4x-19}$.

Section 11.3

4. Divergent P-series.

Section 11.4

2. (a) $\sum a_n$ diverges (b) $\sum a_n$ could be convergent or divergent

Section 11.5

6. Converges by the Alternative Series Test **14.** Divergent by the Test for Divergence

Section 11.6

4. $\sum a_n$ converges absolutely as $\sum |a_n|$ coverges by the Comparison Test as $b_n = \frac{1}{n^3}$.

6. $\sum a_n$ converges conditionally as $\sum |a_n|$ diverges by the Comparison Test with $b_n = \frac{1}{n}$ and $\sum a_n$ converges by AST

38. Conditionally convergent as $\sum |a_n|$ diverges by the Integral Test and $\sum a_n$ converges by AST

Chapter 11 Review

6. Converges to 0 **8.** Converges to 0 **14.** Converges by Alt. Series Test

Additional Homework Problems:

A7:

7. (ab) $V = \pi \int_0^\infty x\, e^{-2x}\, dx = \pi \lim_{t\to\infty} \int_0^t xe^{-2x}\, dx = \frac{\pi}{4}$

8. (ab) $V = \pi \int_1^5 \frac{1}{x-1}\, dx = \pi \lim_{t\to 1^+} \int_t^5 \frac{1}{x-1}\, dx = \infty$

A11:

1. (a) Series diverges by The Comparison Test with $b_n = \frac{1}{\sqrt{n}}$

(b) Series converges by The Comparison Test wirh $b_n = \frac{4}{n^{3/2}}$

(c) Series converges by The Comparison or the Limit Comparison Tests with $b_n = \frac{4^n}{5^n} = \left(\frac{4}{5}\right)^n$

(d) Series diverges by The Divergence test or by The Limit Comparison Tests with $b_n = \frac{5^n}{4^n} = \left(\frac{5}{n}\right)^n$