A5: Integrals

1. A particle moves along a line with velocity function $\mathbf{v}(t) = t - \dfrac{8}{t^2}$, where $\mathbf{v}$ is measured in centimeters per second.

   (a) Find the displacement during the interval [1, 4]

   (b) Find the distance traveled during the interval [1, 4]

A7: Techniques of Integration

1. A particle moves along a line with velocity function $\mathbf{v}(t) = (t-1)e^{-t}$, where $\mathbf{v}$ is measured in meters per minute.

   (a) Find the displacement during the interval [0, 2]

   (b) Find the distance traveled during the interval [0, 2]

2. The base of a solid is the region R bounded by the curve $y = \sin x$ and the lines $y = x$ and $x = \pi/2$.

   (a) Sketch the base R.

   (b) If the cross-sections of the solid perpendicular to the x-axis are isosceles right triangles with hypotenuse in the base. Express the volume of the described solid as a definite integral and then find the volume.

3. The region R in the xy-plane is bounded by the curves $y = 2\cos x$, $y = \tan x$ and the lines $x = 0$, $x = \pi/4$.

   (a) Sketch the region R.

   (b) Find the area of the region R.

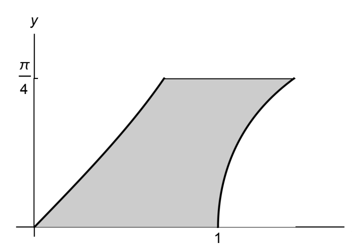
   (c) Find the volume of the solid with region R as its base if its cross-sections perpendicular to the x-axis are squares.

   (d) Find the volume of the solid obtained by rotating the region R about the x-axis.

4. The region D (given in the picture) in the xy-plane is bounded by the curves $y = \arcsin x$, $y = \operatorname{arcsec} x$ and the lines $y = 0$, $y = \pi/4$.

   (a) Find the volume of the solid with region D as its base if its cross-sections perpendicular to y-axis are squares.

   (b) Find the volume of the solid obtained by rotating the region D about y-axis.

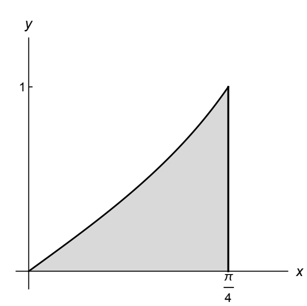


5. The region D (given in the picture) in the xy-plane is bounded by the curves $y = \tan x$, $y = 0$, and $x = \dfrac{\pi}{4}$.

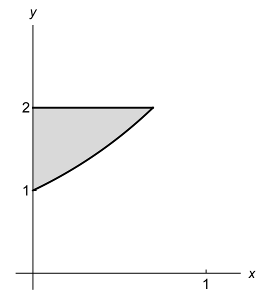
   (a) Find the volume of the solid with region D as its base if its cross-sections perpendicular to x-axis are isoscele right triangles with base in the base.

   (b) Find the volume of the solid obtained by rotating the region D about x-axis.

   (c) Find the volume of the solid obtained by rotating the region D about $y = 1$.



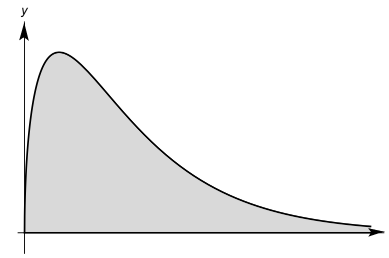
6. The region R (given in the picture) in the xy-plane is bounded by the curves $y = e^x$, $y = 2$, and $x = 0$.



(a) Set up the integral to find the volume of the solid obtained by rotating the region R about y-axis. Don't evaluate it.

(b) Set up the integral to find the volume of the solid obtained by rotating the region R about $x = 1$. Don't evaluate it.

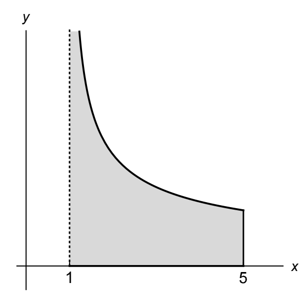
7. Consider the region $D = \{(x, y) \mid x \geq 0,\ 0 \leq y \leq \sqrt{x}\, e^{-x}\}$ as shown in the picture. A solid S is generated by revolving the region D about x-axis.



(a) Write the volume of the solid first as an improper integral and then as a limit of proper definite integrals.

(b) Find the volume of the solid if it is finite. Otherwise, state that it is infinite.

8. Consider the region $D = \{(x, y) \mid 1 < x \leq 5,\ 0 \leq y \leq \dfrac{1}{\sqrt{x-1}}\}$ as shown in the picture. A solid S is generated by revolving the region D about x-axis.



(a) Write the volume of the solid first as an improper integral and then as a limit of proper definite integrals.

(b) Find the volume of the solid if it is finite. Otherwise, state that it is infinite.

A11: Infinite Sequences and Series

1. Determine whether the series converges or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3 + \sin n}{\sqrt{n}}$ **(b)** $\displaystyle\sum_{n=1}^{\infty} \frac{3 + \sin n}{n\sqrt{n}}$ **(c)** $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{3^n + 5^n}$ **(d)** $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{3^n + 4^n}$

2. If $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n}}{4^n (2n)!}$

(a) Find $f'(x)$. Simplify and give your answer in a summation notation.

(b) Evaluate $\displaystyle\int f(x)\, dx$. Simplify and give your answer in a summation notation.

3. If $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{5^n (x-4)^{n+1}}{(n+3)(n+1)!}$

(a) Find $f'(x)$. Simplify and give your answer in a summation notation.

(b) Evaluate $\displaystyle\int f(x)\, dx$. Simplify and give your answer in a summation notation.