**Math 1021 Final Exam Review** (2.2, 2.3, 4.2, 4.7, 5.1-5.4 inclusive, 9.1, 5.5, 5.6 and 6.1-6.6 inclusive, 6.8, 7.1, 7.3, 7.4, 7.5, 8.1, 8.2,8.5, and 8.6)

NOTE THIS IS NOT ALL ENCOMPASSING. THERE MIGHT BE TYPES OF PROBLEMS ON THE TEST THAT ARE NOT ON THIS REVIEW. You must know how to do any of the types of homework problems that were assigned. **Any problem similar to a sample problem or a homework problem may appear on the test. You are also responsible for the examples worked out in each assigned section in the textbook even though they are not done in class.**

Please review problems from Test 1, Test 2 and Test 3 review packets as well.

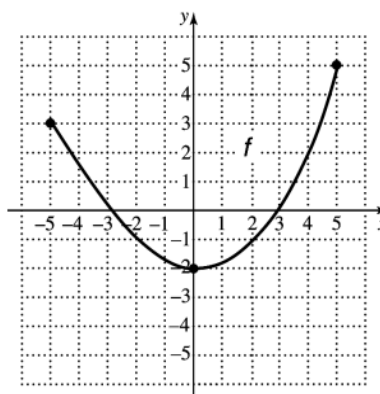**1.** Determine whether the correspondence is a function. Justify your answer.

Domain *Range*

$$6 \longrightarrow -6$$
$$7 \longrightarrow -7$$
$$3 \longrightarrow -3$$
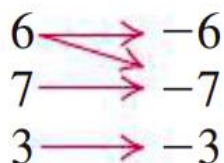
2.

For the graph of function *f* at right, determine
a) $f(4)$;
b) the domain;
c) all *x*-values such that $f(x) = -1$;
and d) the range.

3. Find the domain. Express answers in interval notation.

(a) $f(x) = x^2 - 2x + 1$ **(b)** $g(x) = \dfrac{7}{5-x}$ **(c)** $T(x) = |2x - 3|$

(d) $F(x) = \dfrac{x^3 - x^2 + x + 2}{x^2 + 12x + 35}$ **(e)** $Q(x) = \sqrt{5 - 3x}$ **(f)** $g(x) = \dfrac{1}{\sqrt{x}}$

4. If $f(x) = -5x + 2$ and $g(x) = x^2$. Find

(a) $(f + g)(x)$ (b) $(g - f)(x)$ (c) $(f \cdot g)(-9)$ (d) $(\frac{f}{g})(x)$

(e) $(fog)(x)$ (f) $(gof)(x)$

5. Multiply the following polynomials.

(a) $(3x^3y^5)(-4x^4y^6)$ (b) $(2x^3 - 3x^2 + x + 5)(2x^2 + x - 1)$

(c) $(2x + 3y)^2$ (d) $(x + 2)(x - 2)(x^2 + 4)$

(e) $(2x + 3y)(2x - 3y)$ (f) $-4x^5(2x^3 - 3x^2 + x + 5)$

(g) $(2x - 3y)^3$

6. Factor completely.

(a) $x^2 - 10x + 25$ (b) $4x^2 + 12x + 9$ (c) $9x^2 - 4$

(d) $6x^2 + 10x - 3x - 5$ (e) $x^2 + 4x - 12$

(f) $2x^4 - 16x$ (g) $2x^2 + x - 3$ (h) $12x^2 + 7x - 10$

(i) $2xy^3 - 22xy^2 + 48xy$ (j) $81x^2 - 36x + 4$ (k) $54x^6y - 16y$

7. Perform the indicated operations and simplify your answers.

(a) $\dfrac{x}{x-3} + \dfrac{3}{3-x}$

(b) $\dfrac{1}{4x^2} - \dfrac{2x+1}{3x^3} + \dfrac{3}{12x}$

(c) $\dfrac{y-3}{y^2-4} - \dfrac{y+2}{y^2-4y+4} - \dfrac{2}{2-y}$

(d) $\dfrac{y^2-10y+9}{y^2-1} \cdot \dfrac{y+4}{y^2-5y-36}$

(e) $\dfrac{x+1}{x-x^2} \cdot \dfrac{x^2-2x+1}{x^2-1}$

(f) $\dfrac{4x^2-4x+1}{2x^2+5x-3} \div \dfrac{2x^2-3x-2}{2x^2+7x+3}$

(g) $\dfrac{4x^2-9y^2}{8x^3-27y^3} \div \dfrac{4x^2+12xy+9y^2}{4x^2+6xy+9y^2}$

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8. *Simplify:*

(a) $\dfrac{\frac{x^2}{y^2}-1}{\frac{x}{y}+1}$

(b) $\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$

(c) $\dfrac{\frac{x^2+8x+12}{x^2-5x-14}}{\frac{x^2-x-12}{x^2-2x-15}}$

(d) $\dfrac{\frac{1}{z^2}-\frac{1}{w^2}}{\frac{1}{z^3}+\frac{1}{w^3}}$

9. Divide

(a) $2x^2 + x - 1 \overline{\smash{)}2x^3 - 3x^2 + x + 5}$

(b) $(6x^3 - x^2 - 10) \div (3x + 4)$ `

(c) $(3x^4 + 2x^3 - 11x^2 - 2x + 5) \div (x^2 - 2)$

In 10-13 Solve the equations:

10. $\dfrac{2}{x^2-9} - \dfrac{3}{x-3} = \dfrac{1}{x+3}$

11. $\dfrac{3x+5}{x^2+3x+2} = \dfrac{1}{x+2} + \dfrac{2}{x+1}$

12. $\sqrt{3x-2} = 5$

13. $\sqrt{2x+1} - \sqrt{x+4} = 1$

14. Write each in simplest radical form where appropriate. Variables represent positive values.

(a) $\sqrt{12x^3y^5z^2}$

(b) $\sqrt[3]{\dfrac{8a^7}{27b^3}}$

(c) $\sqrt[3]{2}\ \sqrt[4]{3}$

(d) $\dfrac{\sqrt[4]{w^3}}{\sqrt[3]{w^2}}$

(e) $\sqrt[4]{\dfrac{81x^{21}}{16x^{13}}}$

15. Rationalize the denominator in each of the following.

(a) $\dfrac{5}{\sqrt{5x}}$

(b) $\dfrac{x}{\sqrt[3]{x}}$

(c) $\dfrac{1}{\sqrt{x}-1}$

(d) $\dfrac{\sqrt[3]{2y^4}}{\sqrt[3]{6x^4}}$

(e) $\dfrac{\sqrt{a}-\sqrt{b}}{\sqrt{b}-\sqrt{a}}$

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16. Express the following in terms of rational exponents.

(a) $\left(\sqrt{(x+1)^3}\right)^5$ (b) $\left(\sqrt{(x-1)(x-2)}\right)^3$ (c) $\sqrt{\sqrt[3]{\sqrt{x}}}$

17. Simplify the following, Express in terms of positive exponents. Variables represent positive values.

(a) $\left(27x^3\right)^{2/3}$ (b) $\left(16x^8 y^{-4}\right)^{1/4}$ (c) $\left(\dfrac{x^{-1/3}y^{1/2}}{x^{-1/4}y^{1/3}}\right)^6$

(d) $3^{-5/2}a^{4/5}b^{-7/3}$ (e) $\sqrt[6]{(-4x)^2}$

18. Perform the indicated operations and simplify if possible.

(a) $3\sqrt{a^4}+4\sqrt[3]{8a^6}$ (b) $\sqrt{6b}-\sqrt{24b^3}$ (c) $\left(3-\sqrt{x}\right)\left(3+\sqrt{x}\right)$

(d) $\left(2\sqrt{a}-3\sqrt{b}\right)^2$ (e) $2\sqrt[3]{y}\left(4\sqrt[3]{y}-2\sqrt[3]{y^2}\right)$ (f) $\sqrt[3]{18y^3}\sqrt[3]{4x^2}$

(g) $5\sqrt[3]{16y^4}+7\sqrt[3]{2y}$ (h) $\dfrac{\sqrt[4]{(x-1)^3}}{\sqrt{x-1}}$

19. David can paint the outside of a house in 12 hr. Bill can paint the same house in 9 hr. How long would it take them working together?

20. The current of the Gold River is 6 mph. A boat travels 50 mi downstream in the same time that it takes to travel 30 mi upstream. What is the speed of the boat in still water?

21. A toy rocket is shot vertically into the air from a launching pad 8 feet above the ground with an initial velocity of 48 feet per second. The height h, in feet, of the rocket above the ground at t seconds after launch is given by the function $h(t) = -16t^2 + 48t + 8$.

How long will it take the rocket to reach its maximum height? What is the maximum height?

22. A rectangular parking lot with a straight road as one side is to be fenced on the other three sides by 1000 ft of aluminum fencing. If the area of the lot is to be maximized, what should be its length and width? [Hint: write the area of the lot as $ax^2 + bx + c$.]

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Solve the following equations. **Include any complex solutions.**

23. $11x = 2x^2 + 12$ **24.** $4x^2 = 8x$ **25.** $25x^2 - 9 = 0$ **26.** $x^2 - 12 = 0$

27. $x^2 - 10x - 3 = 0$ **28.** $2x^2 + 1 = 4x$ **29.** $y^{1/2} - 3y^{1/4} + 2 = 0$

30. $x^6 - 6x^3 + 5 = 0$ **31.** $2x^2 - 6x + 3 = 0$ (**Hint: Use complete the square**)

**32.** Use the intermediate value theorem to determine, if possible, whether the function f has at least one real zero between a and b.

$$f(x) = x^4 - 6x^2 - 1; \quad a = 5, b = 6$$

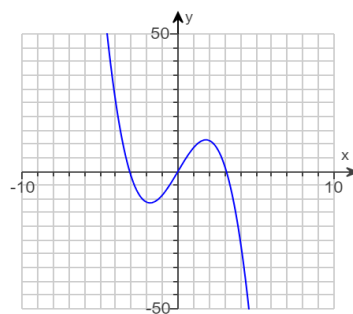**33.** For the function $f(x) = x^{15} - 2x^8 + 7x - 3$, state:

a) the maximum number of real zeros that the function can have;

b) the maximum number of x-intercepts that the graph of the function can have;

c) the maximum number of turning points that the graph of the function can have.

34. Determine visually whether the graph is symmetric with respect to the x-axis, the y-axis, or the origin.

a.
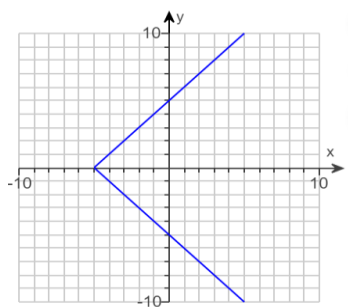
b.

35. For the function, $f(x) = x^2 - 5x + 6$

   a. Find the vertex.

       b. Find the axis of symmetry

       c. Determine whether there is a maximum or minimum value and find that value.

       d. Graph the function

Solve the following inequalities by testing intervals. State your answers in interval notation. Graph your solutions.

36. $x^2 + 21 > 10x$ **37.** $-x^2 - 2x + 3 \geq 0$ **38.** $\dfrac{x-2}{x+4} \leq 0$ **39.** $\dfrac{x^2-x-12}{x^2-4} \geq 0$

40. $x^3 + 3x^2 - x - 3 \geq 0$

In **41-44**, use your knowledge of asymptotes and intercepts to match the equation with one of the graphs (a)-(d) that follows. List the vertical and horizontal asymptotes, if any.

a)

b)

c)

d)
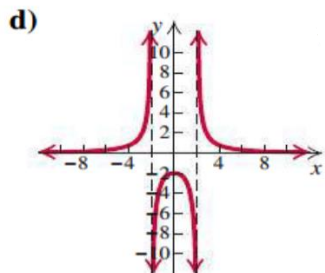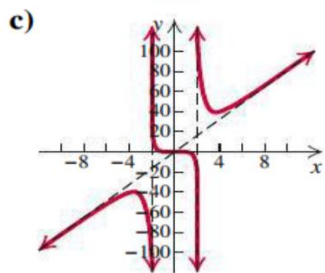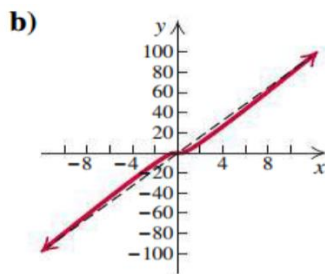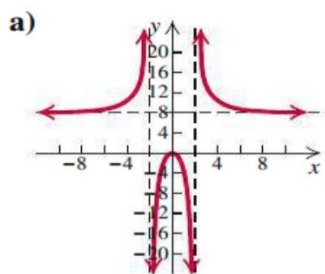
41. $f(x) = \dfrac{8}{x^2-4}$ **42.** $f(x) = \dfrac{8x^2}{x^2-4}$ **43.** $f(x) = \dfrac{8x^3}{x^2-4}$ **44.** $f(x) = \dfrac{8x^3}{x^2+4}$