Math 1021 Review for Test 3 (6.8, 7.1, 7.3, 7.4, 7.5, 8.1, 8.2 and 8.5)

NOTE THIS IS NOT ALL ENCOMPASSING. THERE MIGHT BE TYPES OF PROBLEMS ON THE TEST THAT ARE NOT ON THIS REVIEW. You must know how to do any of the homework problems that were assigned. Any problem similar to a sample problem or a homework problem may appear on the test. You are also responsible for the examples worked out in each assigned section in the textbook even though they are not done in class.

Solve the following equations. Include any complex solutions.

1. \(11x = 2x^2 + 12\)  
2. \(4x^2 = 8x\)  
3. \(25x^2 - 9 = 0\)  
4. \(x^2 - 12 = 0\)
5. \(x^2 - 10x - 3 = 0\)  
6. \(2x^2 + 1 = 4x\)

By completing the square, find all real and complex solutions to the following equations:

7. \(x^2 - 6x - 3 = 0\)  
8. \(2x^2 - 6x + 3 = 0\)
9. \(x^2 - 4x + 5 = 0\)  
10. \(2x^2 + 4x = -5\)

Perform the indicated operation. Please state your answers in the form \(a + bi\) where \(a\) and \(b\) are real numbers.

11. \((-2 + 3i) - (-3 - 2i)\)  
12. \((-2 + 3i)(-3 - 2i)\)  
13. \(-\frac{3 - 2i}{-3 + 2i}\)

Solve the following equations. Include any complex solutions.

14. \(x^4 + 3x^2 = 10\)  
15. \(y^{1/2} - 3y^{1/4} + 2 = 0\) "check required"

16. \(x - 3\sqrt{x} - 4 = 0\) "check required"  
17. \(x^6 - 6x^3 + 5 = 0\)

18. Use the discriminant to determine the type of solution for each of the following quadratic equations.

   a) \(-3x^2 + 2x + 1 = 0\)  
   b) \(-4x^2 + 6x - 5 = 0\)
19. Use the given graph to find each of the following:

a) x-intercepts;   b) y-intercept;

b) the zeros of the function;  d) domain and range;

c) the axis of symmetry;  f) the vertex;

g) the maximum or minimum value of the function;

h) relative maxima or minima of the function and the values of x where they occur

i) is the function odd, even or neither;

j) the intervals of x on which the function is increasing or decreasing

20. Determine visually whether the graph is symmetric with respect to the x-axis, the y-axis, or the origin.

a.   b.
21. Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial function as constant, linear, quadratic, cubic or quartic \( \alpha(t) = 1 - 2t - t^3 \)

22. Find the zeros of the polynomial function and state the multiplicity of each. \( f(x) = \left( x - \frac{3}{2} \right)(x + 3)^2(x - 2)^5 \)

23. Use the intermediate value theorem to determine, if possible, whether the function \( f \) has at least one real zero between \( a \) and \( b \).
\[ f(x) = x^4 - 6x^2 - 1; \quad a = 5, b = 6 \]

24. For the function \( f(x) = x^{15} - 2x^8 + 7x - 3 \), state:
   a) the maximum number of real zeros that the function can have;
   b) the maximum number of \( x \)-intercepts that the graph of the function can have;
   c) the maximum number of turning points that the graph of the function can have.

25. Use the leading-term test to match the function \( f(x) = x^5 + \frac{1}{7}x - 6 \) with one of the given graphs.

26. A toy rocket is shot vertically into the air from a launching pad 8 feet above the ground with an initial velocity of 48 feet per second. The height \( h \), in feet, of the rocket above the ground at \( t \) seconds after launch is given by the function \( h(t) = -16t^2 + 48t + 8 \).
How long will it take the rocket to reach its maximum height? What is the maximum height?
27. A rectangular parking lot with a straight road as one side is to be fenced on the other three sides by 1000 ft of aluminum fencing. If the area of the lot is to be maximized, what should be its length and width? [Hint: write the area of the lot as $ax^2 + bx + c$.]

In 28 & 29, find the axis of symmetry and vertex of the parabola. Find the $x$-intercepts and the $y$-intercept of the parabola. Graph the parabola clearly labeling the vertex, the axis of symmetry and the intercepts.

28. $y = -x^2 - 2x + 3$  
29. $y = x^2 - 2x - 3$

In 30-33, use your knowledge of asymptotes and intercepts to match the equation with one of the graphs (a)-(d) that follows. List the vertical and horizontal asymptotes, if any.

30. $f(x) = \frac{8}{x^2 - 4}$  
31. $f(x) = \frac{8x^2}{x^2 - 4}$  
32. $f(x) = \frac{8x^3}{x^2 - 4}$  
33. $f(x) = \frac{8x^3}{x^2 + 4}$

34. Find the domain. Express answers in interval notation.
   a) $g(x) = \frac{7}{5-x}$  
   b) $F(x) = \frac{x^3-x^2+x+2}{x^2+12x+35}$