MATH 1042 **Review Problems for Test 2** **Fall 2021**

Text: **James Stewart**, *Calculus, Early Transcendentals*, **8**th Edition, Cengage Learning

Chapter 7: Techniques of Integration

7.3: 1, 4, 7, 12, 13

7.4: 1, 3, 5, 19, 22, 23, 28, 65

7.8: 1, 2 (in problems 1 and 2, also express each improper integral as the limit of proper integrals), 5, 7, 9, 17, 19, 21, 27, 29, 41, 50

Chapter 7 Review: True or False: 6. **Exercises:** 71

Chapter 11: Infinite Sequences and Series

11.1: 23, 27, 29, 30, 31, 32, 33, 41, 44, 47, 49, 55, 56

11.2: 3, 4, 15, 22, 23, 26, 29, 31, 33, 37, 38, 39, 44, 46, 47, 57, 59

11.3: 17, 21

11.4: 1, 2, 5, 6, 7, 9, 10, 11, 13, 15, 17, 19, 21, 25, 28

11.5: 5, 7, 11, 12, 13, 18

11.6: 2, 3, 4, 5, 6

Chapter 11 Review: Exercises: 29

- You can memorize the integrals that are used in partial fraction problems, namely

$$\int \frac{1}{x+a}\,dx = \ln|x+a| + C, \qquad \int \frac{1}{(x+a)^n}\,dx = -\frac{1}{n-1}\cdot\frac{1}{(x+a)^{n-1}} + C \quad (n>1),$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C, \qquad \int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\ln(x^2+a^2) + C.$$

- You can use the following limits. Here, $a > 0$ and b is any constant.

$$\lim_{n\to\infty} n^{1/n} = 1, \qquad \lim_{n\to\infty}\left(1+\frac{b}{n}\right)^n = e^b, \qquad \lim_{n\to\infty}\frac{b^n}{n!} = 0, \qquad \lim_{x\to\infty}\frac{x^b}{e^x} = 0, \qquad \lim_{x\to\infty}\frac{\ln x}{x^a} = 0.$$

- When using the Integral Test, you must state that the integrand $f(x)$ is positive and decreasing. The book says that f also needs to be continuous, but this is actually not necessary, so you don't need to mention continuity.

- When using the Integral Test or Alternating Series Test, you need to know that a certain function/sequence is decreasing. If the function is the reciprocal of an increasing function, you may treat this as obvious. If it is not obvious that the function is decreasing, you should prove it. For example "$b_n = \frac{1}{n^2+1}$ is obviously decreasing" would be acceptable, but "$b_n = \frac{n}{n^2+1}$ is obviously decreasing" would not. You could prove that $\frac{n}{n^2+1}$ is decreasing by checking that $\frac{d}{dx}\frac{x}{x^2+1} \le 0$ for all $x \ge 1$.

- When using a series convergence/divergence test, you must state the name of the test and explain clearly why it applies. For example, don't just say "$\sum_{n=1}^{\infty}\frac{n}{n+1}$ diverges because $\lim_{n\to\infty}\frac{n}{n+1} = 1$." Instead, say "$\sum_{n=1}^{\infty}\frac{n}{n+1}$ diverges by the Test for Divergence because $\lim_{n\to\infty}\frac{n}{n+1} = 1 \ne 0$."

- When using the Limit Comparison Test, you may use the notation $a_n \sim b_n$ as a shorter way to express the statement $\lim_{n\to\infty}\frac{a_n}{b_n} = 1$. If this limit is obvious, you do not need to prove it. For example, if you're trying to apply the Limit Comparison Test to $\sum_{n=1}^{\infty}\frac{n+1}{\sqrt{n^3+5n^2+1}}$, you can write $\frac{n+1}{\sqrt{n^3+5n^2+1}} \sim \frac{n}{\sqrt{n^3}} = \frac{1}{n^{1/2}}$ without further explanation.